Diversification effects between stock indices

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Abstract

During the World Financial Crisis it became obvious that classical models of portfolio theory significantly under-estimated risks, especially with regard to stocks. Instabilities of correlations and volatilities, the relevant parameters characterizing risk, led to over-estimation of diversification effects and consequently to under-estimation of risks. In this article, we analyze diversification effects concerning stocks during different market periods of the previous decade. We show that parameters and risks significantly change with market periods and find that the impact of fluctuations and estimation errors is 5 times larger for volatilities than for correlations. Moreover, it turns out that diversification between sectors is more efficient than diversification between countries.

JEL - Classification: C 52, G 11, G 32
JEL - Key words: Model Evaluation, Portfolio Optimization, Risk Management

1 Introduction

Efficient selection of stocks and portfolio optimization are central tasks of the financial sector but are also important for private investors. In this context, strategic asset allocation aims to share a given amount of money *optimally* between different asset classes, considering the crucial parameters of expected return and possible loss. Of particular importance is the diversification between different stock indices.

The model by *Markowitz* (1952) represents a milestone in development of modern theories in the area of asset allocation and portfolio optimization and was rewarded the Nobel price in

Economics in 1990. According to this model, any investor should put his money into efficient portfolios only, i.e. portfolios which have the smallest risk for a given return defined by the investor, or portfolios having a maximal return for a predefined acceptable risk. The risk of the portfolio is given by its variance, i.e. the standard deviation of its overall value. Correlations between the assets may decrease the risk for the overall portfolio significantly compared to investments into single assets.

As shown in numerous works strategic asset allocation makes up for the majority of performance of an investment. *Brinson/Hood/Beebower* (1986) and *Brinson/Singer/Beebower* (1991) quantify the influence as 90% to 94%, while *Ibbotson/Kaplan* (2000) give values between 82% and 88%. Both demonstrating significance of strategic asset allocation. Additional factors, such as timing and strategy realization, are only of minor importance.

Reliable estimation of the relevant parameters, i.e. return, variance and correlation, is of major importance for optimal portfolio selection and therefore future success of the investment. Different studies show that return is the most important parameter in the Markowitz model. *Chopra and Ziemba* (1993) demonstrate that, for mean tolerated risk levels, wrong return estimators have an 11 times larger impact than wrong risk estimators. Analogously, *Kallberg and Ziemba* (1984) and *Schäfer and Zimmermann* (1998) demonstrate that estimation problems in the Markowitz-model are mainly related to the return.

The current situation at the financial markets shifts the focus on the risk perspective. Volatilities and correlations strongly increased during the financial crisis[1], as reflected by strongly increased risk numbers. *Zimmermann/Drobetz/Oertmann* (2002) describe this effect as 'Correlation Breakdown'. Obvisiously, volatilities and correlations of different assets are positively correlated in the crisis, and the diversification approach does not work - in particular when required to prevent losses.

In this paper, we empirically analyze the effects of changing parameters of stock indices during different market periods and give several examples. To this end, the resulting risk numbers for different market periods are compared. From the results we draw conclusions on stability of diversification effects in classical portfolio theory. We are able to show that risks are significantly underestimated, especially if historical mean values are used as parameter estimators. Therefore, reliable diversification requires the introduction of alternative models and methods. We find that the impact of fluctuations and estimation errors is 5 times larger for volatilities than for correlations. Additionally, we determine how these effects influence diversification between countries and between sectors, demonstrating that diversification effects are more stable between the latter.

[1] This work defines volatility as the standard deviation.

2 Classical portfolio theory

The model by *Markowitz* (1952) represents a milestone in development of modern theories in the area of asset allocation and portfolio optimization. It assumes the existence of N assets with normally distributed return r_i for the ith asset. Optimal selection of the portfolio weights $(\omega_1, \omega_2, \cdots, \omega_N)$ is intended where ω_i is the fraction which is invested into asset i.

Up to now, the Markowitz model is broadly used by many investors to optimize portfolios. For many applications it is required that $\omega_i \geq 0$ and $\sum_{i=1}^{N} \omega_i = 1$. The crucial parameters for portfolio selection are the expected return of the portfolio (r_P) and the risk of the portfolio, which is defined by the standard deviation (σ_P).

According to Markowitz-theory efficient portfolios, which are attractive investments, should have a combination (r_P, σ_P), which is not dominated by a portfolio with smaller standard deviation for the same return or a portfolio with a larger return for the same standard deviation. Usually, a portfolio is chosen in such a way that one of the parameters is given and the other one is optimized accordingly. This implies that either a minimum return r_{min} or a maximum volatility σ_{max} has to be pre-defined, resulting in the following optimization problems:

(**OP1**)

$$min \qquad \sigma_P \tag{1}$$

$$\text{s.t.} \qquad r_P \geq r_{min} \tag{2}$$

(**OP1'**)

$$max \qquad r_P \tag{3}$$

$$\text{s.t.} \qquad \sigma_P \leq \sigma_{max}. \tag{4}$$

The consideration of correlation effects between different stocks by this procedure offers the advantage that investments into assets seem to be disadvantagous on the first sight, but may, nevertheless, decrease the overall risk of the portfolio. This is e.g. illustrated by a portfolio containing 80% of an asset with an expected return of 5% and a volatility of 3% and 20% of a more risky asset having an expected return of 10% and a standard deviation of 6%. This combination results in an expected portfolio return of 6%. If both assets are not correlated, the overall volatility of the portfolio is only 2.7%. For a correlation of -0.5, it even decreases to 2.4%, i.e. the expected return of the overall portfolio is larger than the expected return of the more secure asset. Moreover, the risk is significantly smaller than for each single asset, if risk is measured by the standard deviation.

Figure 1 illustrates the effect of different correlations and portfolio weights ($\omega_i \in [0, 1]$) for both assets showing returns and volatilities of the portfolio.

Figure 1 insert here.

The assumptions within this model are that the returns are normally distributed and that the parameters of the assets, i.e. returns, correlations and standard deviations, can be reliably estimated. Moreover, it is assumed that the parameters do not change during the investment period. In the previous years, the reliable estimation of parameters became significantly more difficult: On the one hand, it became obvious that correlations and variances depend on time so that both tend to increase when markets decrease and vice versa. On the other hand, there are strong indications that volatilities and correlations depend on each other as it is shown by *Frennberg and Hansson* (1993), *Zimmermann/Drobetz/Oertmann* (2002) and *Andersen/Bollerslev/Diebold/Ebens* (2001) .

3 Correlation Breakdown

In recent discussions concerning correlations and volatilities in risk management and hedging, the term 'Correlation Breakdown' was introduced and describes the phenomenon that correlations and volatilities tend to increase, if the market decreases and also the other way round. Moreover, there is a strong positive relation between correlations and standard deviations. Thus, diversification effects are particularly overestimated during nervous market periods for which they are of high importance. Hence, the permanent changing pattern of correlations complicates selection of an optimal risk strategy.

The stock market crash in October 1987 and the 2008 financial crisis revealed, that the structure of correlations reflects extreme situations on markets. In both cases correlations strongly increased to a high level remaining constantly high for a certain period.

Meric/Meric (1997) confirm this situation from a European perspective: Average correlations between 13 European stock markets increased from 0.37 before the crash in 1987 to a value of 0.5 afterwards. *Rey* (2000) describes similar events: Average correlations based on data from Switzerland, USA, UK, Canada, Germany, Italy, France and Japan increased from 0.40 measured from January 1973 to December 1986 to 0.55 between January 1988 and December 1999. During October 1987, the average correlation between international stock markets was, according to *Rey* (2000), even 0.68 . A result by *Longin and Solnik* (1995) generally confirms that correlations increase and that also volatilities and correlations are stronger connected when volatility is on a high level.

These results make it necessary for investors to have a critical look on the idea of diversification: Assumptions, which should minimize the overall risk, collapse exactly when markets decrease. Hence, regarding the two great financial crisis of the last decade, it is questionable if classical portfolio theory is able to generate reliable risk estimators.

4 Empirical analysis

This section analyzes the development of correlation structures and volatilities of stock markets during four different phases of the last decade: The complete period of analysis covers March, 31st 1999 to February, 26th 2010. Additionally, two bear markets (dotcom crisis[2], financial crisis[3]) and a bull market[4] are analyzed separately. Figure 2 clarifies the temporal sequence of these periods.

Figure 2 insert here.

4.1 Data base

Monthly final values of representative stock indices are used to determine the relevant parameters of each asset class differing between subsectors (10) and country indices (5). Especially the following stock indices are taken into account for analysis:

- EURO STOXX OIL & GAS
- EURO STOXX BASIC MATERIALS
- EURO STOXX INDUSTRIALS
- EURO STOXX CONSUMER GOODS
- EURO STOXX HEALTH CARE
- EURO STOXX CONSUMER SERVICES
- EURO STOXX TELECOM
- EURO STOXX UTILITIES
- EURO STOXX FINANCIALS
- EURO STOXX TECHNOLOGY
- MSCI EMERGING MARKETS
- MSCI USA
- MSCI JAPAN
- STOXX EUROPE 50
- MSCI WORLD

[2]31.03.2000 to 31.03.2003
[3]30.04.2008 to 31.03.2009
[4]30.04.2003 to 31.03.2008

4.2 Calculation of relevant parameters

In this section, we describe the calculation of the relevant parameters. We investigate for each index a ($a \in \{1, \ldots, m\}$) continous monthly returns. These are determined as

$$r_a(j) = \ln\left(\frac{\text{index at the end of the j-th month}}{\text{index at the end of the (j-1)th month}}\right).$$

The expected average annual return is[5] $\hat{R}_a(J) = 12 \cdot \overline{r_a(J)}$, where $\overline{r_a(J)}$ represents the average monthly return in the respective period. The corresponding months are summarized by the index set J. From the returns $r_a(j)$ for asset class a follows the estimator for the variance of returns

$$\hat{\sigma}_a^2(J) = 12 \cdot \left[\frac{1}{n-1}\sum_{j \in J}\left(r_a(j) - \overline{r_a(J)}\right)^2\right],$$

where n is the number of months in the respective period. Volatility is calculated as the square root of the variance. Analogously, we determine estimators for the correlation between two asset classes a and b ($a, b \in \{1, \ldots, m\}$)

$$\hat{\rho}_{a,b}(J) = \frac{12}{n-1}\sum_{j \in J}\left(\frac{r_a(j) - \overline{r_a(J)}}{\sqrt{\hat{\sigma}_a^2(J)}}\right) \cdot \left(\frac{r_b(j) - \overline{r_b(J)}}{\sqrt{\hat{\sigma}_b^2(J)}}\right)$$

and the estimator for the corresponding covariance

$$\hat{\sigma}_{a,b}^2(J) = \sqrt{\hat{\sigma}_a^2(J) \cdot \hat{\sigma}_b^2(J)} \cdot \hat{\rho}_{a,b}(J).$$

The estimated return $\hat{R}_a(J)$ and variance $\hat{\sigma}_a^2(J)$ yield a parametric estimation of the 99%-Value-at-Risk of an asset class with the 1%-quantile of the standard normal distribution $q_{0.01}$=-2.326 as

$$\text{VaR}_a(99\%)(J) = \hat{R}_a(J) - 2.326 \cdot \sqrt{\hat{\sigma}_a^2(J)}.$$

The $\text{VaR}_a(99\%)$ can be split into a component $\text{VaR}_a(99\%)_{\text{ex}}$, which is given by the expected return (respectively the corresponding estimator) and a 'stochastic' component, $\text{VaR}_a(99\%)_{\text{stoch}} = -2.326 \cdot \sqrt{\hat{\sigma}_a^2(J)}$ which is calculated from the (estimated) volatilities. As shown in the next section correlations also influence the Value-at-Risk of a portfolio because they are required to calculate the overall volatility of a portfolio. Based on the estimated parameters of the different asset classes it is possible to calculate return and risk of a portfolio using the portfolio weights $(\omega_1, \ldots, \omega_n)$. For a period J, the expected return is given by

$$\hat{r}_P(J) = \sum_{a=1}^{m} \omega_a \hat{R}_a(J),$$

[5]For sake of simplicity, the following characteristic numbers, especially volatilities and Value-at-Risks, are given for an one-year investment period.

and its variance is

$$\hat{\sigma}_P^2(J) = \sum_{a,b=1}^{m} \omega_a \omega_b \hat{\sigma}_{a,b}^2(J).$$

Finally, we determine the 99%-Value-at-Risk $\mathrm{VaR}(99\%)_{P(J)}$ of the portfolio over a period J as

$$\mathrm{VaR}(99\%)_{P(J)} = \hat{r}_P(J) - 2.326 \cdot \sqrt{\hat{\sigma}_P^2(J)}.$$

Hence, the stochastic component is

$$\mathrm{VaR}(99\%)_{\mathrm{stoch}} = -2.326 \cdot \sqrt{\hat{\sigma}_P^2(J)}.$$

4.3 Parameters during different market periods

Tables 1 resp. 2 and 3 resp. 4 summarize correlations and volatilities for different market periods sorted by sectors respectively countries showing strong fluctuations of volatility over time. Comparison of parameters during the bull market and the financial crisis, which followed immediately afterwards, shows an alarming increase of volatilities by a factor of 1.5 to 3. Only exemptions from this are the TELECOM and HEALTH CARE sectors. While volatilities in the TELECOM sector strongly increased during the dotcom-crisis they remained on a constant level for the HEALTH CARE sector over the complete observation period.

Surprisingly, it can be observed that the average correlation between all sectors remained constant over all periods, i.e. there was no 'Correlation Breakdown' even not during the financial crisis. Thus, diversification between sectors appears to remain stable even during crisis. For country indices, this result does not turn out to be true. Average correlations were on a constant level until upset of the crisis. More precise, the average correlation between countries increased by 0.21 during financial crisis. Even the smallest value was 0.84. This shows a clear 'Correlation Breakdown'.

For correlations between single indices, even higher fluctuations can be observed. This turns out to be true for sectors as well as for countries, e.g. the correlation between TELECOM and HEALTH CARE decreased between bull market and financial crisis by 0.47, while correlation between Japan and the US increased by 0.44[6].

These surprising results demonstrate that diversification effects between sectors also work during crisis but not between countries, where structures of correlations change. Thus, diversification within the asset category 'stocks' between countries seems to be impossible and the true risks are significantly larger than expected.

Tables 1 bis 4 insert here.

4.4 Effects of changing parameters on the VaR

To illustrate and to quantify the effects of changing parameters, we consider risk numbers of five different portfolios for each period. Three portfolios reflect sectors whereas two are diversified by countries. A large variability of correlations and volatilities leads to a strongly varying stochastic component ($\mathrm{VaR}_{\mathrm{stoch}}$) of the overall Value-at-Risk (VaR). Since changes in the stochastic component are based on variability of correlations and volatilities, we restrict our analysis to this component as it also represents the effect of diversification which can be achieved for a portfolio. This component on its own leads to a strong change of the overall VaR.

Two of the portfolios use a naive diversification where all indices hold the same share of the

[6]Complete correlation matrices can be obtained at www.quasol.de/publikationen.html.

overall portfolio, one being diversified by sectors and the other one by countries. Also two funds, based on sectors (AriDeka CF, Deka-Institutionell Aktien Europa I (T)), and one fund, based on different countries (Deka-bav Fonds), are analyzed. Exact diversification of the portfolios is given in Tables 6 and 7. For sake of simplicity, we assume that the asset categories contained in the portfolio are perfectly reflected by the index. Hence, we obtain realistic estimations for the behavior of risk numbers of real portfolios although they are not exactly replicated, which is not in the scope of this work. Furthermore, we assume an investment of 100.000.000€ to provide the VaR in €.

Tables 6 and 7 insert here.

Tables 8-12 show the stochastic component $\mathrm{VaR}_{\mathrm{stoch}}$ for all portfolios. Our results strongly indicate that by solely varying correlations and volatilities the VaR is dramatically fluctuating. Thus, the VaR increased by a factor of ≈ 2 $\mathrm{VaR}_{\mathrm{stoch}}$ for all portfolios upon exchange of the bull market parameters by values holding for the financial crisis. Even during the dotcom-crisis, the risk was significantly larger than during the bull market.
Comparing sector-based to country-based portfolios, fluctuations are smaller for the first. During bull market, the risk for sector based portfolios was slightly larger whereas it was smaller during the financial crisis.

Tables 8-12 insert here.

We performed another data analysis to investigate whether the changes in risk are caused by changing correlations or by changing volatilities (or to find out which are their respective contributions). Here, we assumed for all market periods the average volatilities solely changing correlation matrices. Hence, changes of the covariance matrix result from changing correlations. Based on these covariance matrices, volatilities of the naively diversified portfolios were determined for all market periods. Results are given in table 5.
For sector indices it was shown that the increased risk is completely explained by increased volatilities. If the volatility only changed due to changes of the correlation matrix, it would remain constant over all market periods being consistent to results in the prior section, showing that the average correlation did not change. Considering country based indices, it turned out that the volatility increased by 2% during the financial crisis due to increased correlations. If we also took the changes in volatilities into account, the increase would be about 12% between countries, i.e. the effect of changing volatilities on the risk is about five times larger than that of changing correlations. To draw a conclusion, fluctuations in volatilities have a significantly stronger impact on diversification effects than changes in correlations.

Tables 5 insert here.

5 Conclusion and outlook

Results of the last section show that risks of the individual portolios differ strongly in dependence of the time period which is used for parameter estimation. These differences in the risk are important to consider for institutional as well as for private investors. Thus, it is of great interest to analyze how these risks can be minimized or at least be appropriately measured. It is demonstrated that correlations and volatilities cannot be estimated simply from historical data due to large estimation errors in some cases.
To illustrate this effect Figure 3 represents the different temporal evolution of estimators of correlation between the sectors TELECOM and FINANCIALS, where the correlation was estimated from historical data using different moving averages.

Figure 3 insert here.

The illustration shows that fluctuations of the estimators decrease with an increase of the time period used for analysis. This implies the problem that a long time period leads to very inflexible estimators, due to its strong smoothing of the results, so that changes of the parameters are only considered by the result after a long time span. Thus, during a bull market, the risk is overestimated whereas it is underestimated during crisis. If only short time periods are used for estimation this may lead to drastic estimation errors because of strong variability of the estimators. Immediately after the bullmarket, the estimators strongly deviate from the true values having occured during the financial crisis.
In current research, there exist different approaches to deal with changing parameters, e.g. it is possible to use modern parameter estimators which are more accurate and flexible than the historical mean value. Time series models are of special interest with regard to this because they adapt to changing data structures in a very flexible way. In addition to the history of the time series, GARCH models also consider their own history and the history of the estimation error (*McNeil/Frey/Embrechts* (2005)).
An additional approach for the timely recognition of parameter changes is testing for structural breaks, i.e. changes in parameters which define a time series. This topic is in the scope of current research and there are very promising results, such as timely recognition of a structural break, which enables introduction of measures to react to a changing market situation.

For more details please refer to *Krämer/Tameze* (2007), *Qu/Perron* (2007) and *Krämer/van Kampen* (2009).

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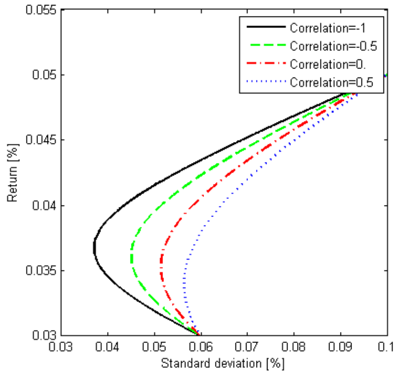

Figure 1: Efficiency frontiers for portfolios consisting of two stocks with returns and standard deviations of $(5\%, 3\%)$ and $(10\%, 6\%)$ for different correlations between the stocks.

Table 1: Volatilities during different market periods (sectors)

Index / Period	Total	Dotcom	Bull Market	Financial Crisis
OIL & GAS	18,05%	18,18%	15,08%	26,78%
BASIC MATERIALS	21,54%	24,93%	15,96%	33,02%
INDUSTRIALS	21,27%	22,79%	15,49%	34,09%
CONSUMER GOODS	18,99%	21,50%	14,51%	23,73%
HEALTH CARE	16,19%	19,46%	14,09%	17,91%
CONSUMER SERVICES	18,80%	25,29%	13,48%	20,30%
TELECOM	26,35%	35,98%	14,41%	14,95%
UTILITIES	17,42%	16,76%	12,74%	24,42%
FINANCIALS	24,24%	28,16%	16,09%	42,27%
TECHNOLOGY	31,42%	48,71%	22,05%	36,19%

Table 2: Volatilities during different market periods (countries)

Index / Period	Total	Dotcom	Bull Market	Financial Crisis
EMERGING MARKETS	22,19%	20,91%	15,80%	33,09%
USA	15,05%	18,64%	9,17%	27,11%
JAPAN	19,02%	15,33%	14,58%	31,71%
EUROPE	16,77%	19,30%	11,15%	21,22%
WORLD	15,12%	16,77%	9,74%	28,79%

Figure 2: Schematic illustration of the analyzed periods at the example of the development of the EURO STOXX 50.

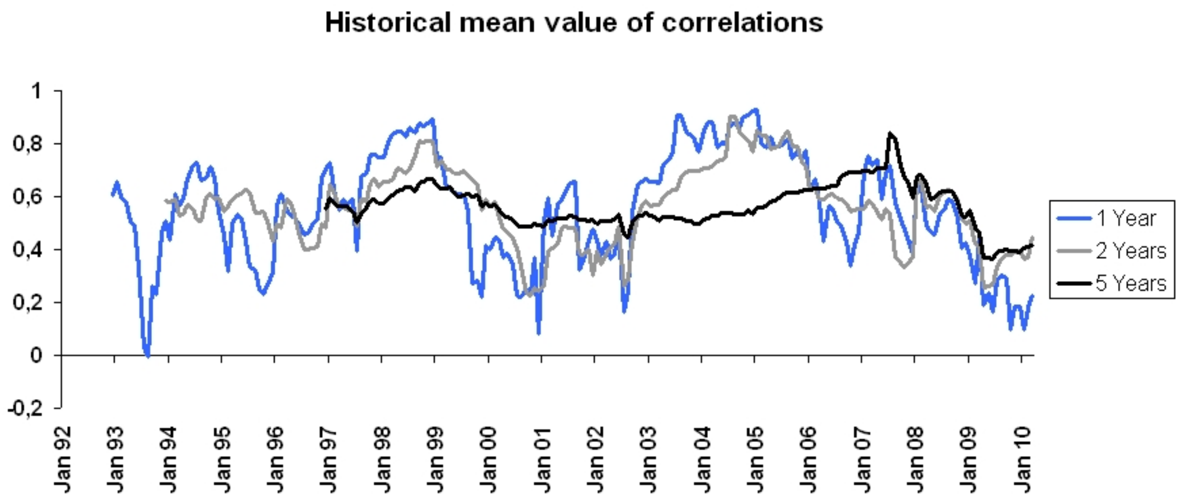

Figure 3: Moving averages of the correlation between TELECOM and FINANCIALS for a history of 1-year, 2-years and 5-years.

Table 3: Average correlation for different market periods (sectors)

Index / Period	Total	Dotcom	Bull Market	Financial Crisis
OIL & GAS	0,53	0,52	0,45	0,53
BASIC MATERIALS	0,66	0,61	0,64	0,64
INDUSTRIALS	0,73	0,68	0,71	0,71
CONSUMER GOODS	0,68	0,69	0,69	0,48
HEALTH CARE	0,46	0,36	0,39	0,47
CONSUMER SERVICES	0,69	0,67	0,69	0,66
TELECOM	0,50	0,40	0,55	0,38
UTILITIES	0,63	0,52	0,65	0,70
FINANCIALS	0,69	0,71	0,69	0,68
TECHNOLOGY	0,63	0,61	0,54	0,67
AVERAGE	0,62	0,58	0,60	0,59

Table 4: Average correlation for different market periods (countries)

Index / Period	Total	Dotcom	Bull Market	Financial Crisis
EMERGING MARKETS	0,65	0,67	0,69	0,87
USA	0,72	0,76	0,71	0,92
JAPAN	0,55	0,48	0,54	0,89
EUROPE	0,70	0,70	0,70	0,84
WORLD	0,78	0,79	0,77	0,93
AVERAGE	0,68	0,68	0,68	0,89

Table 5: Resulting volatilities using average volatilities for single indices and changing correlation matrices by period

Period / Volatility	Sector	Country
Total	17,47%	15,11%
Dotcom	17,03%	15,07%
Financial crisis	17,21%	15,15%
Bull market	17,14%	16,80%

Table 6: Portfolio weights (sectors)

Index / Portfolio	Naiv	AriDeka CF	Deka-Institutionell
OIL & GAS	10,00%	12,32%	15,60%
BASIC MATERIALS	10,00%	10,89%	9,01%
INDUSTRIALS	10,00%	8,49%	4,29%
CONSUMER GOODS	10,00%	14,23%	10,88%
HEALTH CARE	10,00%	13,76%	16,92%
CONSUMER SERVICES	10,00%	6,70%	1,98%
TELECOM	10,00%	7,78%	9,45%
UTILITIES	10,00%	4,31%	5,60%
FINANCIALS	10,00%	19,02%	23,74%
TECHNOLOGY	10,00%	2,51%	2,53%

Table 7: Portfolio weights (countries)

Index / Portfolio	Naiv	Deka-bav Fonds
EMERGING MARKETS	20,00%	0,67
USA	20,00%	44,70%
JAPAN	20,00%	6,20%
EUROPE	20,00%	35,30%
WORLD	20,00%	13,80

Table 8: $\text{VaR}_{\text{stoch}}$ Naive diversification (sectors)

Period / $\text{VaR}_{\text{stoch}}$	In %	In €
Total	-40,64%	-33,4 Mio. €
Dotcom	-48,60%	-38,5 Mio. €
Financial crisis	-52,56%	-40,9 Mio. €
Bull market	-28,57%	-24,9 Mio. €

Table 9: $\text{VaR}_{\text{stoch}}$ AriDeka CF

Period / $\text{VaR}_{\text{stoch}}$	In %	In €
Total	-39,66%	-32,7 Mio. €
Dotcom	-45,71%	-36,7 Mio. €
Financial crisis	-53,62%	-41,5 Mio. €
Bull market	-28,27%	-24,6 Mio. €

Table 10: VaR_{stoch} Deka-Institutionell

Period / VaR_{stoch}	In %	In €
Total	-39,21%	-32,4 Mio. €
Dotcom	-44,89%	-36,2 Mio. €
Financial crisis	-53,98%	-41,7 Mio. €
Bull market	-27,65%	-24,2 Mio. €

Table 11: VaR_{stoch} Naive diversification (countries)

Period / VaR_{stoch}	In %	In €
Total	-35,14%	-29,6 Mio. €
Dotcom	-36,70%	-30,7 Mio. €
Financial crisis	-63,16%	-46,8 Mio. €
Bull market	-24,07%	-21,4 Mio. €

Table 12: VaR_{stoch} Deka-bav Fonds

Period / VaR_{stoch}	In %	In €
Total	-34,08%	-28,9 Mio. €
Dotcom	-40,42%	-33,2 Mio. €
Financial crisis	-57,79%	-43,9 Mio. €
Bull market	-21,72%	-19,5 Mio. €