A new online-test for changes in correlations between assets

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Abstract

We apply a new test to determine whether correlations between assets are constant over time. The test statistic is a suitably standardized maximum of cumulative empirical correlation coefficients. An empirical application to various assets suggests that the test performs well in applications. We also propose a portfolio strategy based on our test which hedges against potential financial crises and show that it works in practice.

Keywords: Correlation, Econometric Modeling, Finance, Portfolio Optimization
JEL: C12, C14, G01, G11

1 Introduction

During the recent financial crisis, capital market volatilities and correlations increased quite dramatically. As a consequence, risk figures increased significantly, diversification effects were overestimated and ultimately, capital was lost. In literature, this phenomenon is sometimes referred to as "Diversification Meltdown" (Campbell et al., 2008) and is well known also from other contexts. Indeed, there is quite a consensus in empirical finance, that correlations among many time series cannot be assumed to remain constant over longer periods of time (Longin and Solnik, 1995; Krishan et al., 2009, among many others). In particular, correlations among stock returns seem to increase in times of crisis (Sancetta and Satchell, 2007). During the crash in 1987 there was a considerable increase in correlations. Meric and Meric, 1997, approved this fact for european stocks as the average correlation between 13 European stock indices raised from 0.37 before the crash to 0.50 afterwards. Similar results can be found in Rey, 2000, among many others. A comparison of the correlations during different market phases in the last ten years, which yields similar results, can be found in Bissantz et al., 2010b, and Bissantz et al., 2010a.

A correlation breakdown has serious consequences for portfolio optimization which is based on diversification effects between several assets. If the relevant parameters (e.g. correlations) change, the optimization is no longer valid and the risk incorrectly estimated. Similar problems occur to applications in risk management or to the valuation of financial instruments. Surprisingly, there is a lack of methods to formally test for changes in correlations or volatilities.

Most existing procedures either require strong parametric assumptions (Dias and Embrechts, 2004), assume that potential break points are known (Pearson and Wilks, 1933; Jennrich, 1970; Goetzmann et al., 2005), or simply estimate correlations from moving windows without giving a formal decision rule (Longin and Solnik, 1995). Only recently, Aue et al. (2009) have proposed a formal test for a change in covariance structure that does not build upon prior knowledge as to the timing of potential shifts. It is based on cumulated sums of second order empirical cross moments (in the vain of Ploberger et al., 1989) and rejects the null of constant covariance structure if these cumulated sums fluctuate too much.

In this paper, we investigate a test proposed by Wied (2009) which focuses on correlations. The test statistic is a suitably standardized maximum of cumulatively calculated empirical correlation coefficients. We analyze the correlation structure between four indices including stocks, bonds and commodities. The test performs very well throughout the whole empirical application and the resulting dates of rejection seem to be reasonable. Moreover, we use the test to derive an investment strategy, which is evaluated by an out-of-sample study.

The paper is organized as follows. First, we give a short example for an application area of our test. After that, we describe the test statistic and summarize the required theory. Finally, we perform several tests based on real data and discuss the results. Cumbersome formulas are given in the appendix.

2 Application Areas

Markowitz, 1952, developed a theory which can be seen as a milestone in modern asset allocation. He assumed that there are N assets with anticipated normally distributed return r_i for the i-th asset. The problem is to find an optimal assignment of portfolio weights $(\omega_1, \omega_2, \ldots, \omega_N)$ with $\omega_i \geq 0$ and $\sum_{i=1}^{N} \omega_i = 1$, where ω_i is the fraction invested in asset i.
The relevant parameters for the optimization are the expected return of the portfolio (r_P) and the risk, which is defined as the portfolio's volatility (σ_P). This procedure depends crucially on the assumptions of normally distributed returns and constant parameters. In the last years there where several results which show that both assumptions fail. There is some evidence that the returns do not follow a normal distribution and variances and correlations of different assets vary over time. Moreover, there are some indications that volatility and correlation of stock/asset returns tend to increase as the market decreases and also the other way round (Frennberg and Hansson, 1993; Zimmermann et al., 2002; Andersen et al., 2001).
As the correlation structure between the assets directly influences σ_P, these results have been alarming. If correlations rise, σ_P increases and hence the risk rises. So, it is crucial to test for changes in the correlation structure and incorporate the results throughout portfolio optimization.
As the variance/covariance approach is also used in various applications in risk management, the same holds true for this application area. Furthermore, the knowledge of the correlation structure is important for the valuation of financial instruments and lies at the heart of the capital asset pricing model and the arbitrage pricing theory (Embrechts et al., 1999).

3 Test Statistic

Let $(X_t, Y_t), t = 1, 2...$, be a sequence of bivariate random vectors with finite first four moments. We allow for some serial dependence. To be more precise, the (X_t, Y_t) are assumed to be near-epoch dependent on a strong mixing or uniform mixing sequence. Variations of the variances are also permitted and for example GARCH-effects are covered by our assumptions. For more details about technical assumptions see Wied (2009).
We want to test whether the correlation between X_t and Y_t,

$$\rho_t = \frac{Cov(X_t, Y_t)}{\sqrt{Var(X_t)}\sqrt{Var(Y_t)}},$$

is constant over time. Our test statistic is

$$\hat{D} \max_{2 \le j \le T} \frac{j}{\sqrt{T}} |\hat{\rho}_j - \hat{\rho}_T|, \tag{1}$$

where

$$\hat{\rho}_k = \frac{\sum_{i=1}^{k}(X_i - \bar{X}_k)(Y_i - \bar{Y}_k)}{\sqrt{\sum_{i=1}^{k}(X_i - \bar{X}_k)^2}\sqrt{\sum_{i=1}^{k}(Y_i - \bar{Y}_k)^2}}$$

with $\bar{X}_k = \frac{1}{k}\sum_{i=1}^{k} X_i,\ \bar{Y}_k = \frac{1}{k}\sum_{i=1}^{k} Y_i$.

The expression $\hat{\rho}_k$ is the empirical correlation coefficient calculated from the first k observations. The test rejects the null hypothesis of constant correlation if the empirical correlations fluctuate too strongly, as measured by $\max_{2 \le j \le T} |\hat{\rho}_j - \hat{\rho}_T|$. The weighting factor $\frac{j}{\sqrt{T}}$ scales down deviations at the beginning, where the $\hat{\rho}_j$ are more variable, and the scalar factor $\hat{D}$ captures the volatilities of X_t and Y_t as well as the dependence of (X_t, Y_t) over time in order to derive the asymptotic null distribution. The factor $\hat{D}$ is cumbersome to write down, but can easily be calculated from the data. The exact formula is given in the appendix. In practice, there are several variants of $\hat{D}$ depending on the choice of kernel and bandwidth which all lead to asymptotically valid tests. In our empirical application, we choose the Bartlett kernel so that $\hat{D}$ is well defined even in small samples. Furthermore, we choose $[\log(T)]$ as bandwidth. After transforming the time scale from $t \in \{2, \ldots, T\}$ to $z \in [0, 1]$, the test statistic can be rewritten as

$$\sup_{0 \le z \le 1} \left| \hat{D} \frac{\tau(z)}{\sqrt{T}} (\hat{\rho}_{\tau(z)} - \hat{\rho}_T) \right|$$

where $\tau(z) = [2 + z(T-2)]$. The asymptotic null distribution is $\sup_{0 \le z \le 1} |B(z)|$, where B is a one-dimensional Brownian bridge.

This distribution is well known, see Billingsley (1968). Using the quantiles of this distribution, we obtain an asymptotic test for our problem. More precisely, we reject the null hypothesis of constant correlation, if

$$\hat{D} \max_{2 \le j \le T} \frac{j}{\sqrt{T}} |\hat{\rho}_j - \hat{\rho}_T| > q_{1-\alpha}, \tag{2}$$

where $q_{1-\alpha}$ is the $(1-\alpha)$-quantile of $\sup_{0 \le z \le 1} |B(z)|$.

4 Empirical Applications

4.1 Historical rejection dates

The test is applied to several assets: two stock indices (S&P 500, DAX), a commodity index (CRB Spot Index) and a government bond index (REX), using daily data (final quote) and

Table 1: Indices under investigation

Index I	Index II	Period of time
S&P	DAX	05.01.1965 - 01.04.2010
S&P	REX	04.01.1988 - 01.04.2010
S&P	CRB	26.05.1981 - 01.04.2010
CRB	DAX	26.05.1981 - 01.04.2010
CRB	REX	04.01.1988 - 01.04.2010
DAX	REX	04.01.1988 - 01.04.2010

the longest available time series for each combination of indices. Table 1 gives all tested combinations and the corresponding time periods. The procedure for the test is as follows. We start at the 20-th available data point and increase the period of time successively for one day. The starting point is due to the fact that approximately 20 data points are required for a reliable estimation of the correlation between two assets. For each of these time intervals the test is applied for $\alpha = 5\%$ and $\alpha = 1\%$, respectively. This procedure is performed until the tests rejects the null hypothesis of constant correlation. Then, the 20-th day after rejection is the new starting point and the procedure is repeated for the remaining time span. This procedure is due to the fact that correlations cannot be assumed to be constant anymore, if the null hypothesis is rejected. A new reliable estimation requires another 20 data points *after* the point in time, where the correlation changed. Otherwise, the estimator would be biased as data of two different phases were mixed.

Tables 2 and 3 give the rejection dates of the null hypothesis for both confidence levels.

The results seem to be reasonable. For stock indices, there are a lot of rejections in 2000, 2003 and 2008. These data mark the beginning of the Dotcom-crisis (2000) and financial crisis (2008), while in 2003 a bull market started. It is worthwile to mention that, in 2008, constant correlations between REX and all other risky assets are rejected until end of September on a 5% level. Moreover, a change in correlation between REX and DAX is detected at the eighth of September 2008, i.e. shortly before Lehman collapsed.

For DAX and REX, the test yields very interesting results. Figure 1[1] shows the average correlation over the corresponding time interval, the rolling 250-day correlation and the rejection dates. Between 1988 and 1998, the correlation is about 0.5, with exception of 1989, which can probably be explained by reunification of Germany. A positive correlation corresponds to the fact that decreasing interest rates lead to increasing stock markets. This is in common with economic theory as cheap money supports the growth of industry. This connection changed dramatically. In the last years, there was a negative correlation between REX and DAX. Since

[1]The complete results for all combinations of assets and more figures can be found at www.quasol.de/publikationen.html.

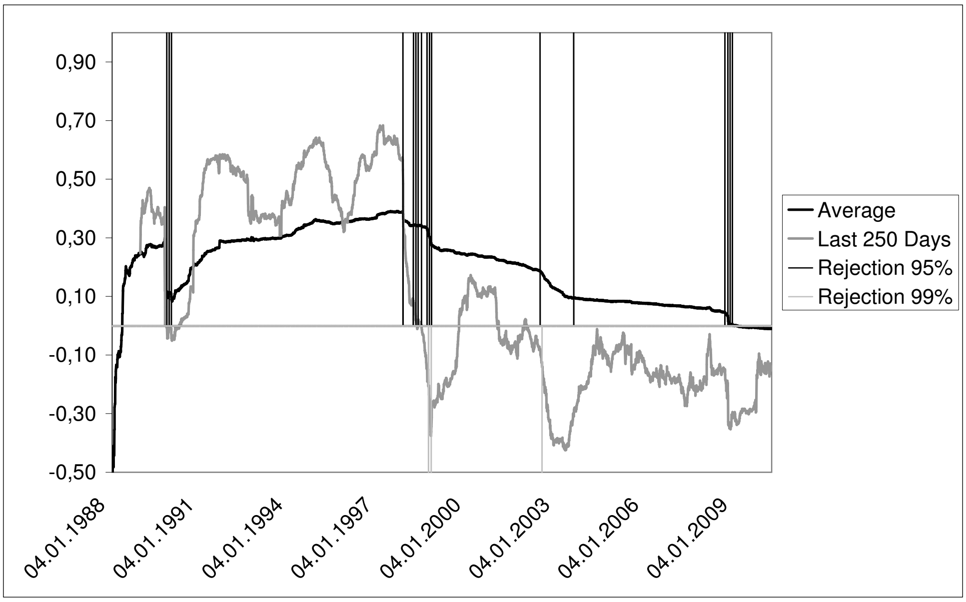
Table 2: Rejection Dates ($\alpha = 5\%$)

S&P & DAX	S&P & REX	S&P & CRB	CRB & DAX	CRB & REX	DAX & REX
11.02.1965	04.08.1998	25.09.1981	17.07.1981	16.06.1988	13.11.1989
28.06.1965	01.09.1998	14.12.1981	10.10.1986	18.07.1988	11.12.1989
13.05.1970	31.01.2000	11.01.1982	21.10.1987	15.08.1988	08.01.1990
22.10.1987	08.03.2000	05.03.1985	24.02.1999	31.01.1989	29.10.1997
23.12.1999	22.12.2000	26.10.1987	25.03.2002	01.03.1989	05.03.1998
20.01.2000	28.08.2002	11.02.1999	22.04.2002	09.09.1998	07.04.1998
22.11.2000	15.10.2002	11.03.1999	28.06.2002	23.09.2008	05.05.1998
20.12.2000	01.08.2003	09.10.2008	17.03.2008		15.06.1998
10.04.2001	11.04.2008		07.07.2008		21.08.1998
14.09.2001	30.09.2008		04.08.2008		18.09.1998
21.10.2002	28.10.2008		01.09.2008		16.10.1998
10.12.2002			10.10.2008		14.06.2002
07.01.2003					01.08.2003
25.03.2003					08.09.2008
22.02.2008					14.10.2008
15.10.2008					11.11.2008
					09.12.2008

Table 3: Rejection Dates ($\alpha = 1\%$)

S&P & DAX	S&P & REX	S&P & CRB	CRB & DAX	CRB & REX	DAX & REX
29.05.1970	09.10.1998	11.07.1986	17.07.1987	18.07.1988	07.09.1998
26.10.1987	08.02.2002	08.08.1986	14.08.1987	15.08.1988	09.10.1998
23.12.1999	30.12.2008	26.10.1987	19.10.1987	02.03.1989	05.07.2002
20.01.2000		29.10.1999	20.04.1999	09.11.1999	
05.12.2000		13.10.2008	17.06.2002	29.09.2008	
03.01.2001			08.10.2008		
21.02.2001					
10.04.2001					
17.09.2001					
17.09.2008					

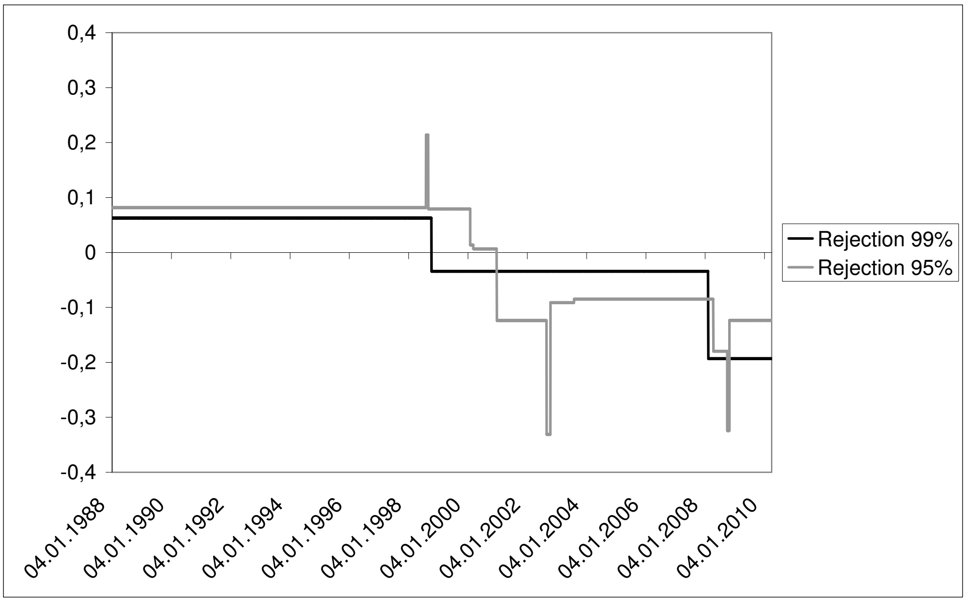
Figure 1: REX and DAX



many crisis occured during the last years, government bonds are bought if the stock markets decrease and the other way round. Hence, a negative correlation between relatively risk free assets (REX) and risky assets (DAX) can be observed.
In general, the correlations fluctuate strongly. Figure 2 exemplarily illustrates the estimated correlation of REX and S&P between different rejection dates. The correlation fluctuates between 0.21 and -0.33 which yields a range of 0.54 in total. This example demonstrates the importance of a reliable test for constant correlation.
Our results show that the chosen confidence level plays an important role for both rejection frequency and rejection dates. Consequently, the confidence level has to be chosen carefully in practical applications. We suggest $\alpha = 1\%$ for a long-run trading strategy, whereas $\alpha = 5\%$ might be more reasonable for risk minimization.

Finally, we return to our suggested testing procedure. It implies that, after a date for which the test rejects the null hypothesis, reliable estimates of the correlation between some assets are not available for approximately one month. In this case, if the Markowitz approach is used, a new optimization leaving out at least one of the formerly used assets has to be performed as it is not advisable to include assets without a reliable estimate of the related risk quantities. In order to avoid losses, we suggest to dismiss the most risky asset (or assets). Using this strategy, at the end of September 2008 no risky asset would have been in a portfolio and a lot of losses

Figure 2: Average correlation between rejection points of REX and S&P



could have been avoided throughout the financial crisis.
As an illustration, Figure 3 shows the development over time of the assets considered here and points of time, where the test rejects for a large part of the pairwise correlations between these assets.

4.2 A Trading Strategy

In order to investigate the possibility to derive trading strategies, which are based on the proposed test, we perform an out of sample study. In this study, we compare two simple strategies. In a first step, Strategy 1 incorporates always the longest time span available in order to calculate the historical return for each of the four assets. In a second step, the capital is uniformly distributed between all assets, whose historical return is positive.
Strategy 2 applies the proposed test. Instead of the longest available time span, the longest available time span since the last detected change in correlation to an other asset is used to calculate the historical return and the volatility. In addition to that, the more risky asset, where the risk is measured by volatility, is not allowed to be bought for 20 days, if a change in correlation is detected between two assets. Finally, the capital is uniformly distributed between all allowed assets, whose modified historical return is positive. Portfolio shiftings are done the day after the test rejected in order to design the study as realistic as possible. Moreover, we

Figure 3: Rejection during financial crisis

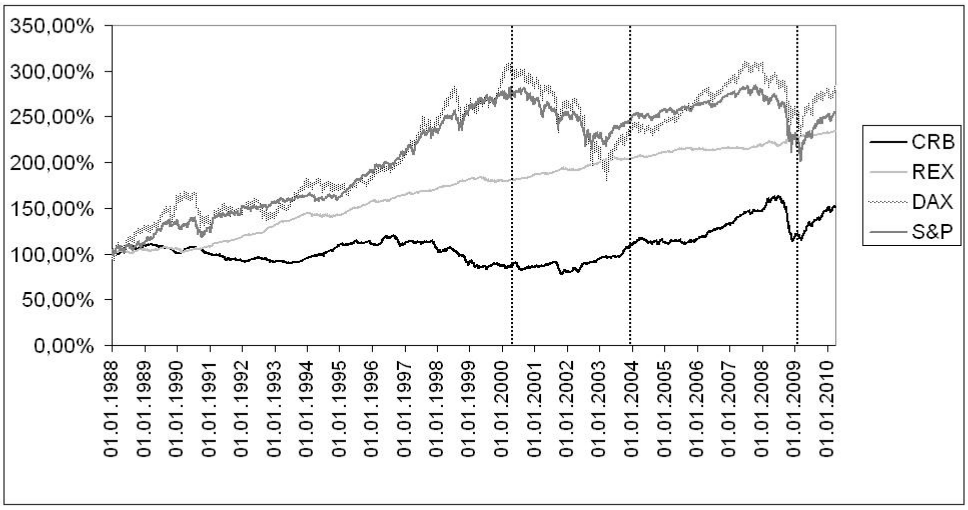


Table 4: Summary statistics for all indices and strategies

Figure	CRB	REX	DAX	S&P	Strategy 1	Strategy 2
Total return	151.91%	234,26%	283,03%	256,19%	247,48%	368,71%
Volatility (1 day)	0,41%	0,21%	1,44%	1,13%	0,66%	0,53%

choose $\alpha = 1\%$ for the test and neglect transaction costs.

The results can be found in Figure 4 and Table 4. The total return of Strategy 2 is higher than the total return of Strategy 1 and all single assets. Moreover, the portfolio development of Strategy 2 is much more stable and only a little money is lost during financial crisis. This result is very remarkable as three risky assets are considered throughout the study.

Figures 5 and 6 show the resulting portfolio weights of the respective strategy over time. Strategy 1 yields an inflexible development of portfolio weights. In contrast to that, the portfolio weights of Strategy 2 change more often. Especially during crises, a lot of fluctuation can be observed. This ensures the good performance of Strategy 2, because most of the downward movement in bear markets is automatically avoided.

If the suggested proceeding is not applicable (e.g. because it is not allowed to dismiss an asset completely), or the correlation is required to determine the Value at Risk or the price of a financial instrument, other strategies have to be found. For example, intra-day data could be used to estimate the correlation (Barndorff-Nielsen and Shephard, 2004) or subjective but conservative assumptions concerning the correlations could be made.

Figure 4: Comparison of trading strategies

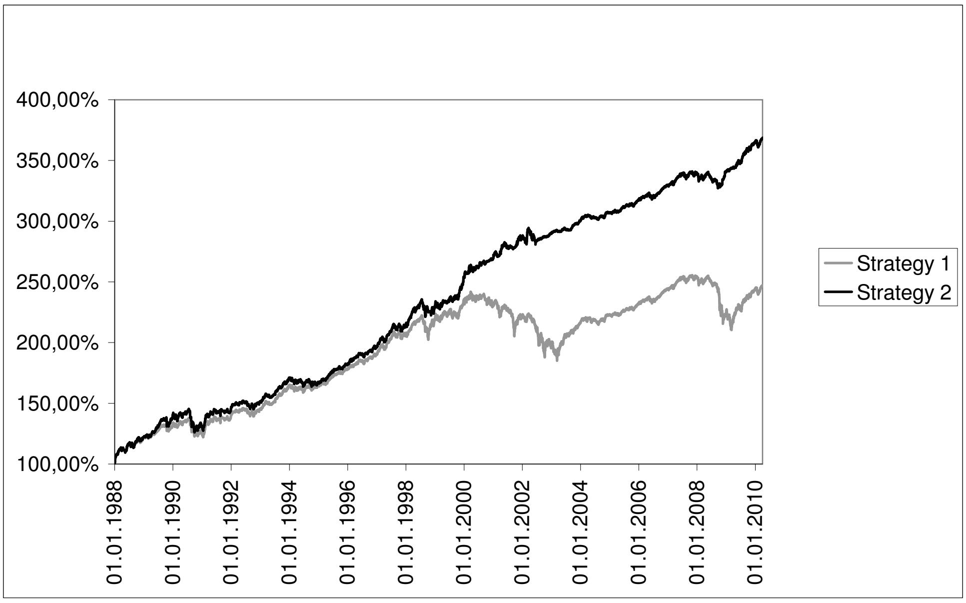


Figure 5: Portfolio weights (Strategy 1)

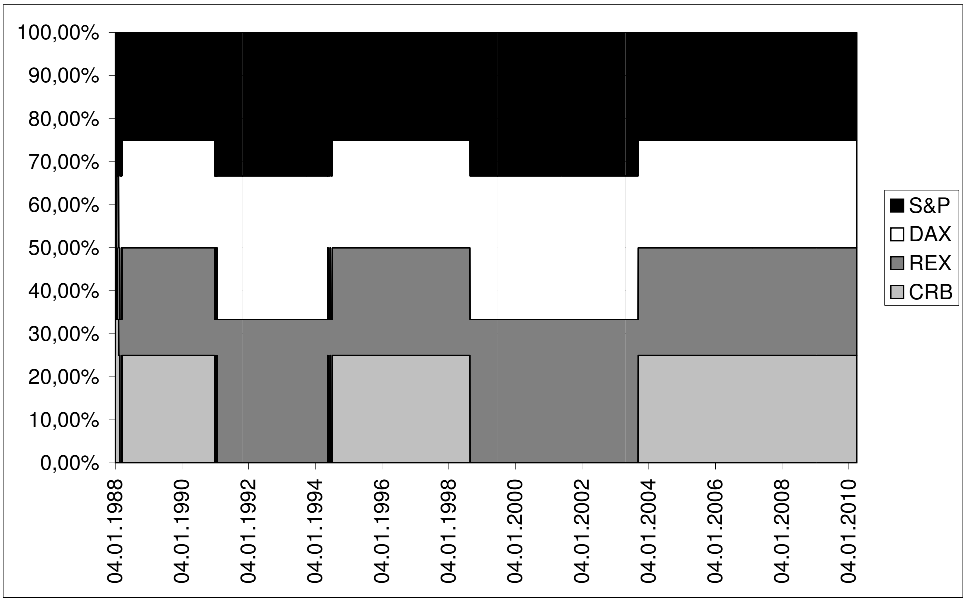
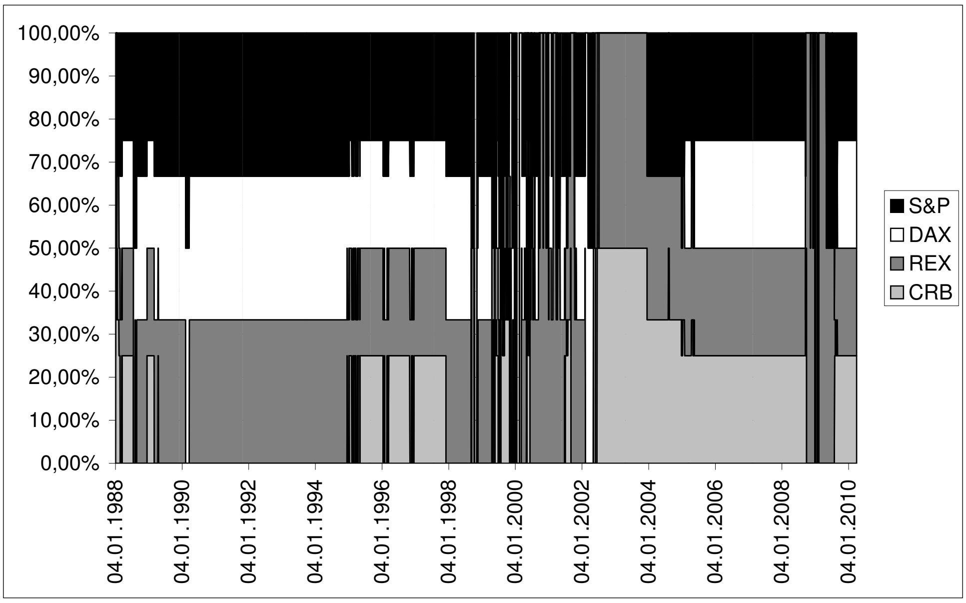


Figure 6: Portfolio weights (Strategy 2)



5 Summary

Current research and developments in finance show the need for tests for changes in market parameters. We investigated the performance of a test which determines whether correlations between assets are constant over time. The test was performed for various assets and a long period of time. The results and rejection dates seem reasonable. Moreover, strategies to use the test for portfolio optimization were suggested. These strategies ensure a more conservative asset management and risk management using the variance/covariance approach. Because of these advantages and its simplicity, the proposed test is interesting for practical investigations.

6 Appendix

The formula $\hat{D}$ from the test statistic (1) is given by

$$\hat{D} = (\hat{F}_1 \hat{D}_{3,1} + \hat{F}_2 \hat{D}_{3,2} + \hat{F}_3 \hat{D}_{3,3})^{-\frac{1}{2}}$$

where

$$\begin{pmatrix} \hat{F}_1 & \hat{F}_2 & \hat{F}_3 \end{pmatrix} = \begin{pmatrix} \hat{D}_{3,1}\hat{E}_{11} + \hat{D}_{3,2}\hat{E}_{21} + \hat{D}_{3,3}\hat{E}_{31} \\ \hat{D}_{3,1}\hat{E}_{12} + \hat{D}_{3,2}\hat{E}_{22} + \hat{D}_{3,3}\hat{E}_{32} \\ \hat{D}_{3,1}\hat{E}_{13} + \hat{D}_{3,2}\hat{E}_{23} + \hat{D}_{3,3}\hat{E}_{33} \end{pmatrix}',$$

$$\hat{E}_{11} = \hat{D}_{1,11} - 4\hat{\mu}_x\hat{D}_{1,13} + 4\hat{\mu}_x^2\hat{D}_{1,33},$$
$$\hat{E}_{12} = \hat{E}_{21} = \hat{D}_{1,12} - 2\hat{\mu}_x\hat{D}_{1,23} - 2\hat{\mu}_y\hat{D}_{1,14} + 4\hat{\mu}_x\hat{\mu}_y\hat{D}_{1,34},$$
$$\hat{E}_{22} = \hat{D}_{1,22} - 4\hat{\mu}_y\hat{D}_{1,24} + 4\hat{\mu}_y^2\hat{D}_{1,44},$$
$$\hat{E}_{13} = \hat{E}_{31} = -\hat{\mu}_y\hat{D}_{1,13} + 2\hat{\mu}_x\hat{\mu}_y\hat{D}_{1,33} - \hat{\mu}_x\hat{D}_{1,14} + 2\hat{\mu}_x^2\hat{D}_{1,34} + \hat{D}_{1,15} - 2\hat{\mu}_x\hat{D}_{1,35},$$
$$\hat{E}_{23} = \hat{E}_{32} = -\hat{\mu}_y\hat{D}_{1,23} + 2\hat{\mu}_x\hat{\mu}_y\hat{D}_{1,44} - \hat{\mu}_x\hat{D}_{1,24} + 2\hat{\mu}_y^2\hat{D}_{1,34} + \hat{D}_{1,25} - 2\hat{\mu}_y\hat{D}_{1,45},$$
$$\hat{E}_{33} = \hat{\mu}_y^2\hat{D}_{1,33} + 2\hat{\mu}_x\hat{\mu}_y\hat{D}_{1,34} - 2\hat{\mu}_y\hat{D}_{1,35} + \hat{\mu}_x^2\hat{D}_{1,44} + \hat{D}_{1,55} - 2\hat{\mu}_x\hat{D}_{1,45},$$

$$\hat{D}_1 = \begin{pmatrix} \hat{D}_{1,11} & \hat{D}_{1,12} & \hat{D}_{1,13} & \hat{D}_{1,14} & \hat{D}_{1,15} \\ \hat{D}_{1,21} & \hat{D}_{1,22} & \hat{D}_{1,23} & \hat{D}_{1,24} & \hat{D}_{1,25} \\ \hat{D}_{1,31} & \hat{D}_{1,32} & \hat{D}_{1,33} & \hat{D}_{1,34} & \hat{D}_{1,35} \\ \hat{D}_{1,41} & \hat{D}_{1,42} & \hat{D}_{1,43} & \hat{D}_{1,44} & \hat{D}_{1,45} \\ \hat{D}_{1,51} & \hat{D}_{1,52} & \hat{D}_{1,53} & \hat{D}_{1,54} & \hat{D}_{1,55} \end{pmatrix} = \sum_{t=1}^{T}\sum_{u=1}^{T} k\left(\frac{t-u}{\gamma_T}\right) V_t V_u{}',$$

$$V_t = \frac{1}{\sqrt{T}} U_t^{***}, \gamma_T = [\log T],$$

$$U_t^{***} = \left(X_t^2 - \overline{(X^2)}_T \quad Y_t^2 - \overline{(Y^2)}_T \quad X_t - \bar{X}_T \quad Y_t - \bar{Y}_T \quad X_tY_t - \overline{(XY)}_T \right)',$$

$$k(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & otherwise \end{cases},$$

$$\hat{\mu}_x = \bar{X}_T, \hat{\mu}_y = \bar{Y}_T, \hat{D}_{3,1} = -\frac{1}{2}\frac{\hat{\sigma}_{xy}}{\hat{\sigma}_y}\hat{\sigma}_x^{-3}, \hat{D}_{3,2} = -\frac{1}{2}\frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x}\hat{\sigma}_y^{-3}, \hat{D}_{3,3} = \frac{1}{\hat{\sigma}_x\hat{\sigma}_y},$$

$$\hat{\sigma}_x^2 = \overline{(X^2)}_T - (\bar{X}_T)^2, \hat{\sigma}_y^2 = \overline{(Y^2)}_T - (\bar{Y}_T)^2, \hat{\sigma}_{xy} = \overline{(XY)}_T - \bar{X}_T\bar{Y}_T,$$

and

$$\overline{(X^2)}_T = \frac{1}{T}\sum_{t=1}^{T} X_t^2, \; \overline{(Y^2)}_T = \frac{1}{T}\sum_{t=1}^{T} Y_t^2,$$
$$\bar{X}_T = \frac{1}{T}\sum_{t=1}^{T} X_t, \; \bar{Y}_T = \frac{1}{T}\sum_{t=1}^{T} Y_t,$$
$$\overline{(XY)}_T = \frac{1}{T}\sum_{t=1}^{T} X_tY_t.$$

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